

**Warsaw University  
of Technology**



**Faculty of Power and  
Aeronautical Engineering**

WARSAW UNIVERSITY OF TECHNOLOGY

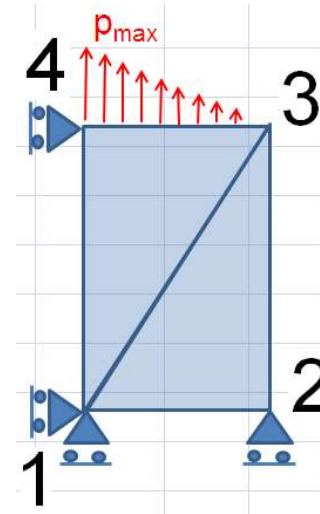
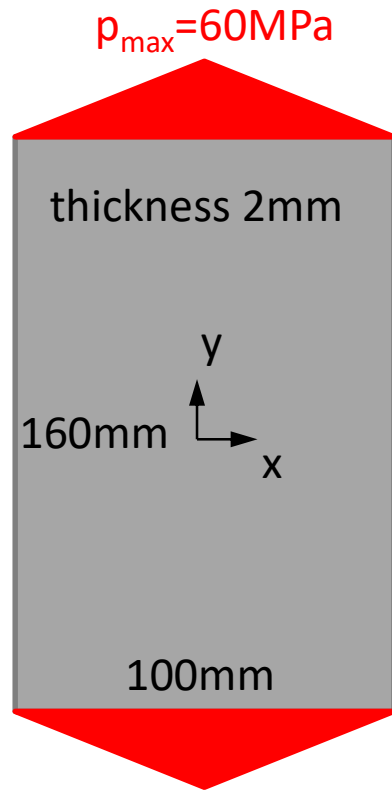
Institute of Aeronautics and Applied Mechanics

# Finite element method (FEM)

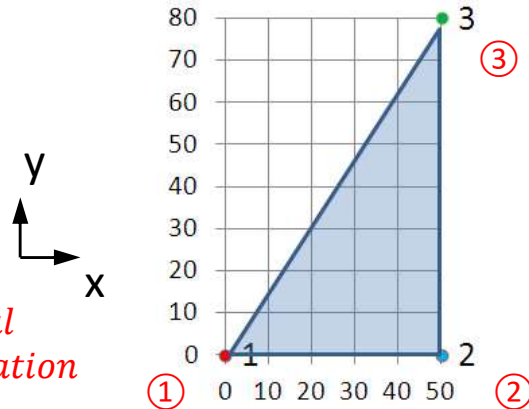
2D Plate modeled by CST finite elements

03.2021

# Model of a rectangular plate



**one-quarter model with two finite elements**



Element	1
E=	7.00E+04 MPa
ni=	0.33333333
he=	2 mm
Ae=	2000 mm <sup>2</sup>

$$a_i = x_j y_k - x_k y_j$$

$$b_i = y_j - y_k$$

$$c_i = x_k - x_j$$

	node	x i	y i	x j	y j	x k	y k	a i	b i	c i
①	1	0	0	50	0	50	80	4000	-80	0
②	2	50	0	50	80	0	0	0	80	-50
③	3	50	80	0	0	50	0	0	0	50

$$B_1 =$$

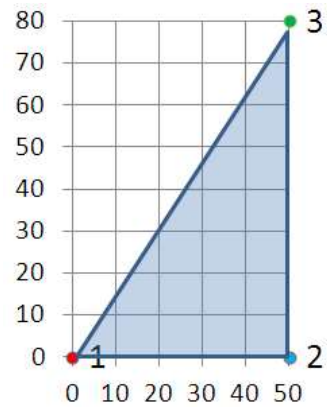
-0.02	0	0.02	0	0	0
0	0	0	-0.0125	0	0.0125
0	-0.02	-0.0125	0.02	0.0125	0

$$B_1^T =$$

-0.02	0	0
0	0	-0.02
0.02	0	-0.0125
0	-0.0125	0.02
0	0	0.0125
0	0.0125	0

$$D =$$

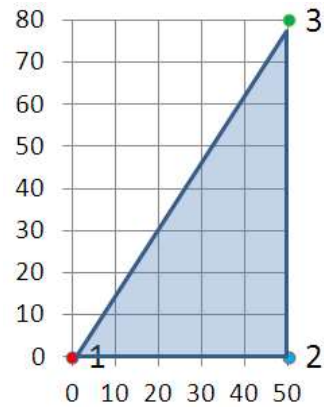
78750	26250	0
26250	78750	0
0	0	26250



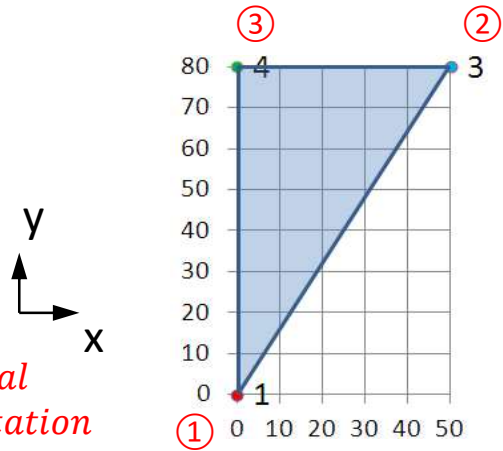
$$B_1^T D B_1 =$$

	31.5	0	-31.5	6.5625	0	-6.5625
	0	10.5	6.5625	-10.5	-6.5625	0
	-31.5	6.5625	35.60156	-13.125	-4.10156	6.5625
	6.5625	-10.5	-13.125	22.80469	6.5625	-12.3047
	0	-6.5625	-4.10156	6.5625	4.101563	0
	-6.5625	0	6.5625	-12.3047	0	12.30469

			u	v	u	v	u	v
			1	1	2	2	3	3
$k_1 =$	u	1	126000	0	-126000	26250	0	-26250
	v	1	0	42000	26250	-42000	-26250	0
	u	2	-126000	26250	142406.3	-52500	-16406.3	26250
	v	2	26250	-42000	-52500	91218.75	26250	-49218.8
	u	3	0	-26250	-16406.3	26250	16406.25	0
	v	3	-26250	0	26250	-49218.8	0	49218.75



		u1	v1	u2	v2	u3	v3	u4	v4
	u1	126000	0	-126000	26250	0	-26250	0	0
	v1	0	42000	26250	-42000	-26250	0	0	0
	u2	-126000	26250	142406.3	-52500	-16406.3	26250	0	0
<b><math>k_1^*</math></b>	v2	26250	-42000	-52500	91218.75	26250	-49218.8	0	0
	u3	0	-26250	-16406.3	26250	16406.25	0	0	0
	v3	-26250	0	26250	-49218.8	0	49218.75	0	0
	u4	0	0	0	0	0	0	0	0
	v4	0	0	0	0	0	0	0	0



local notation

Element	2
E=	7.00E+04 MPa
ni=	0.33333333
he=	2 mm
Ae=	2000 mm <sup>2</sup>
p <sub>max</sub> =	60 MPa

$$a_i = x_j y_k - x_k y_j$$

$$b_i = y_j - y_k$$

$$c_i = x_k - x_j$$

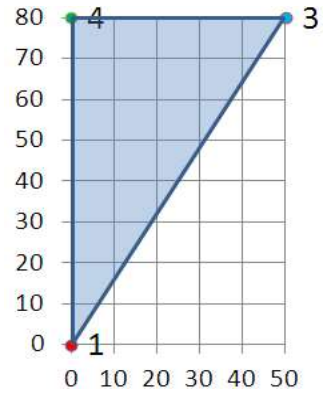
- ①
- ②
- ③

node	x <sub>i</sub>	y <sub>i</sub>	x <sub>j</sub>	y <sub>j</sub>	x <sub>k</sub>	y <sub>k</sub>	a <sub>i</sub>	b <sub>i</sub>	c <sub>i</sub>
1	0	0	50	80	0	80	4000	0	-50
3	50	80	0	80	0	0	0	80	0
4	0	80	0	0	50	80	0	-80	50

$$B_2 = \begin{bmatrix} 0 & 0 & 0.02 & 0 & -0.02 & 0 \\ 0 & -0.0125 & 0 & 0 & 0 & 0.0125 \\ -0.0125 & 0 & 0 & 0.02 & 0.0125 & -0.02 \end{bmatrix}$$

$$B_2^T = \begin{bmatrix} 0 & 0 & -0.0125 \\ 0 & -0.0125 & 0 \\ 0.02 & 0 & 0 \\ 0 & 0 & 0.02 \\ -0.02 & 0 & 0.0125 \\ 0 & 0.0125 & -0.02 \end{bmatrix}$$

$$D = \begin{bmatrix} 78750 & 26250 & 0 \\ 26250 & 78750 & 0 \\ 0 & 0 & 26250 \end{bmatrix}$$

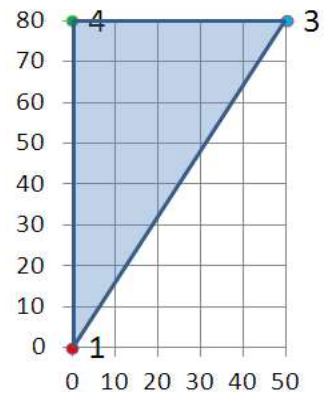


$$\mathbf{B}_2^T \mathbf{D} \mathbf{B}_2 =$$

	4.1015625	0	0	-6.5625	-4.10156	6.5625
	0	12.30469	-6.5625	0	6.5625	-12.3047
	0	-6.5625	31.5	0	-31.5	6.5625
	-6.5625	0	0	10.5	6.5625	-10.5
	-4.1015625	6.5625	-31.5	6.5625	35.60156	-13.125
	6.5625	-12.3047	6.5625	-10.5	-13.125	22.80469

$$\mathbf{k}_2 =$$

		u	v	u	v	u	v	
		1	1	3	3	4	4	
	u	1	16406.25	0	0	-26250	-16406.3	26250
	v	1	0	49218.75	-26250	0	26250	-49218.8
	u	3	0	-26250	126000	0	-126000	26250
	v	3	-26250	0	0	42000	26250	-42000
	u	4	-16406.25	26250	-126000	26250	142406.3	-52500
	v	4	26250	-49218.75	26250	-42000	-52500	91218.75



		u1	v1	u2	v2	u3	v3	u4	v4
	u1	16406.25	0	0	0	0	-26250	-16406.3	26250
	v1	0	49218.75	0	0	-26250	0	26250	-49218.75
	u2	0	0	0	0	0	0	0	0
<b><math>k_2^*</math></b>	v2	0	0	0	0	0	0	0	0
	u3	0	-26250	0	0	126000	0	-126000	26250
	v3	-26250	0	0	0	0	42000	26250	-42000
	u4	-16406.3	26250	0	0	-126000	26250	142406.3	-52500
	v4	26250	-49218.8	0	0	26250	-42000	-52500	91218.75

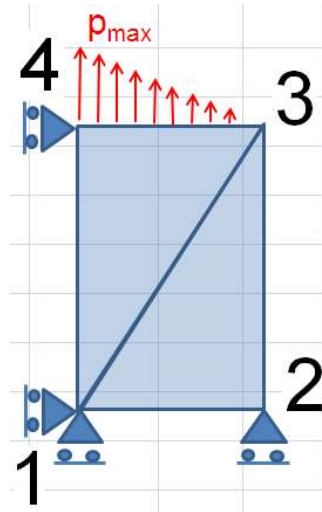


		u1	v1	u2	v2	u3	v3	u4	v4
<b>k<sub>1</sub>*=</b>	u1	126000	0	-126000	26250	0	-26250	0	0
	v1	0	42000	26250	-42000	-26250	0	0	0
	u2	-126000	26250	142406.3	-52500	-16406.3	26250	0	0
	v2	26250	-42000	-52500	91218.75	26250	-49218.8	0	0
	u3	0	-26250	-16406.3	26250	16406.25	0	0	0
	v3	-26250	0	26250	-49218.8	0	49218.75	0	0
	u4	0	0	0	0	0	0	0	0
	v4	0	0	0	0	0	0	0	0

		u1	v1	u2	v2	u3	v3	u4	v4
<b>k<sub>2</sub>*=</b>	u1	16406.25	0	0	0	0	-26250	-16406.3	26250
	v1	0	49218.75	0	0	-26250	0	26250	-49218.75
	u2	0	0	0	0	0	0	0	0
	v2	0	0	0	0	0	0	0	0
	u3	0	-26250	0	0	126000	0	-126000	26250
	v3	-26250	0	0	0	0	42000	26250	-42000
	u4	-16406.3	26250	0	0	-126000	26250	142406.3	-52500
	v4	26250	-49218.8	0	0	26250	-42000	-52500	91218.75

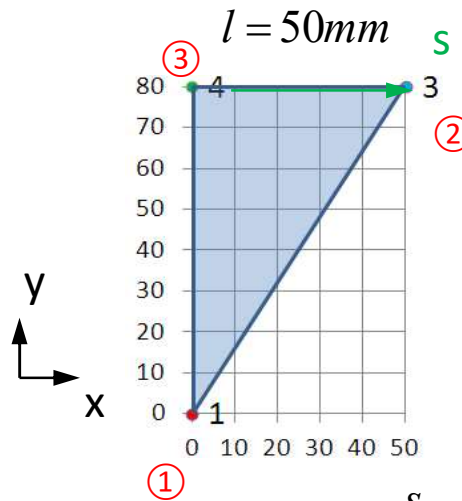
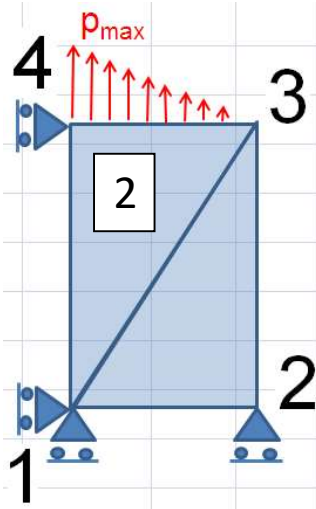


		u1	v1	u2	v2	u3	v3	u4	v4
<b>K=</b> NDOF × NDOF	u1	142406.3	0	-126000	26250	0	-52500	-16406.3	26250
	v1	0	91218.75	26250	-42000	-52500	0	26250	-49218.75
	u2	-126000	26250	142406.3	-52500	-16406.3	26250	0	0
	v2	26250	-42000	-52500	91218.75	26250	-49218.8	0	0
	u3	0	-52500	-16406.3	26250	142406.3	0	-126000	26250
	v3	-52500	0	26250	-49218.8	0	91218.75	26250	-42000
	u4	-16406.3	26250	0	0	-126000	26250	142406.3	-52500
	v4	26250	-49218.8	0	0	26250	-42000	-52500	91218.75



**Boundary conditions**

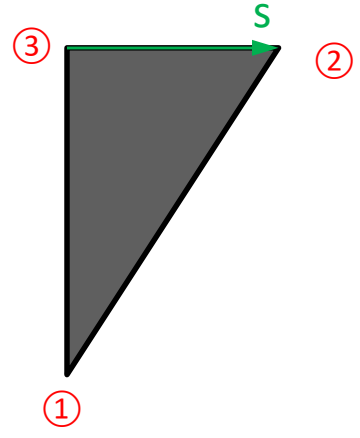
		u1	v1	u2	v2	u3	v3	u4	v4	
<b>K=</b> NDOF × NDOF	u1	142406.3	0	-126000	26250	0	-52500	-16406.3	26250	u1 = 0
	v1	0	91218.75	26250	-42000	-52500	0	26250	-49218.75	v1 = 0
	u2	-126000	26250	142406.3	-52500	-16406.3	26250	0	0	
	v2	26250	-42000	-52500	91218.75	26250	-49218.8	0	0	v2 = 0
	u3	0	-52500	-16406.3	26250	142406.3	0	-126000	26250	
	v3	-52500	0	26250	-49218.8	0	91218.75	26250	-42000	
	u4	-16406.3	26250	0	0	-126000	26250	142406.3	-52500	u4 = 0
	v4	26250	-49218.8	0	0	26250	-42000	-52500	91218.75	



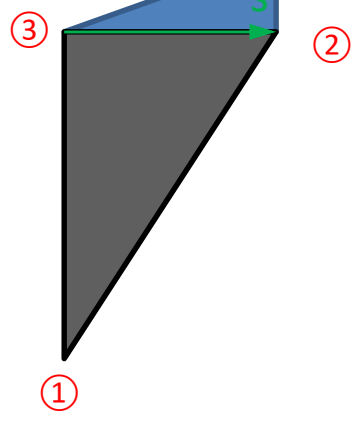
$$\underline{p} = \begin{bmatrix} p_x & p_y \end{bmatrix} = \begin{bmatrix} 0 & p(s) \end{bmatrix} = \begin{bmatrix} 0 & p_{\max} \left(1 - \frac{s}{l}\right) \end{bmatrix}$$

$p_{\max} = 60 \text{MPa}$

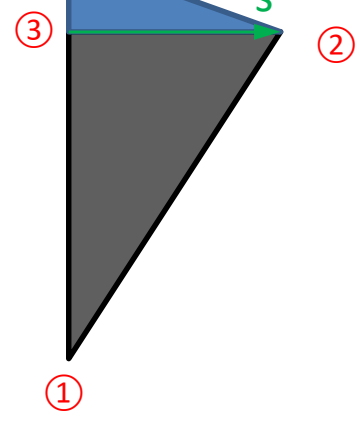
$$N_1(s) = 0$$



$$N_2(s) = \frac{s}{l}$$



$$N_3(s) = 1 - \frac{s}{l}$$



$$\begin{aligned} \underline{F}^p \Big|_2 &= h \int_0^l \underline{p}_x(s), \underline{p}_y(s) \begin{bmatrix} N_1(s) & 0 & N_2(s) & 0 & N_3(s) & 0 \\ 0 & N_1(s) & 0 & N_2(s) & 0 & N_3(s) \end{bmatrix} ds = \\ &= \underline{F}_1^p, \underline{F}_2^p, \underline{F}_3^p, \underline{F}_4^p, \underline{F}_5^p, \underline{F}_6^p \Big|_2 \end{aligned}$$

$$F_1^p = h \int_0^l p_x(s) N_1(s) ds = h \int_0^l 0 \cdot 0 ds = 0$$

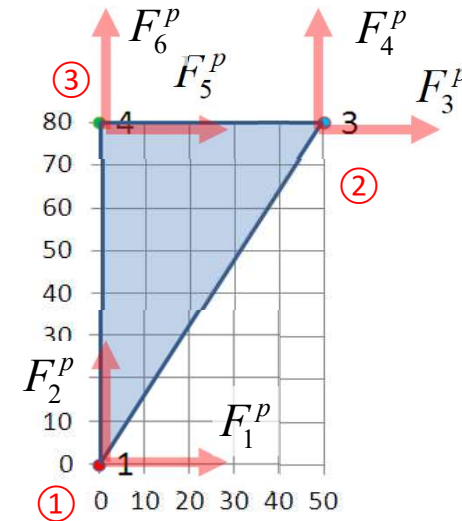
$$F_2^p = h \int_0^l p_y(s) N_1(s) ds = h \int_0^l p_{\max} \left(1 - \frac{s}{l}\right) \cdot 0 ds = 0$$

$$F_3^p = h \int_0^l p_x(s) N_2(s) ds = h \int_0^l 0 \cdot \frac{s}{l_1} ds = 0$$

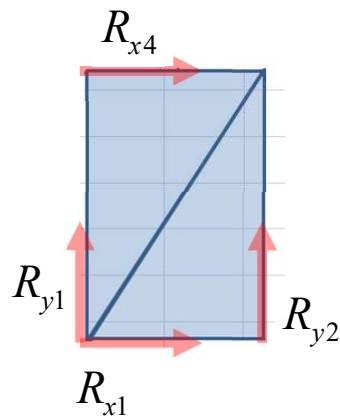
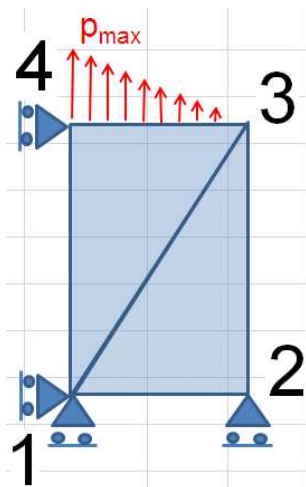
$$F_4^p = h \int_0^l p_y(s) N_2(s) ds = h \int_0^l p_{\max} \left(1 - \frac{s}{l}\right) \cdot \frac{s}{l} ds = \frac{1}{6} p_{\max} lh = 1000N$$

$$F_5^p = h \int_0^l p_x(s) N_3(s) ds = h \int_0^l 0 \cdot \left(1 - \frac{s}{l}\right) ds = 0$$

$$F_6^p = h \int_0^l p_y(s) N_3(s) ds = h \int_0^l p_{\max} \left(1 - \frac{s}{l}\right) \cdot \left(1 - \frac{s}{l}\right) ds = \frac{1}{3} p_{\max} lh = 2000N$$



	0
	0
	0
<b>F<sup>e</sup> =</b>	0
	0
	1000
	0
	2000

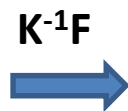


	0		Rx1		Rx1
	0		Ry1		Ry1
	0		0		0
$F^e =$	0		$F^n =$ Ry2		$F =$ Ry2
	0		0	NDOF $\times$ 1	0
	1000		0		1000
	0		Rx4		Rx4
	2000		0		2000

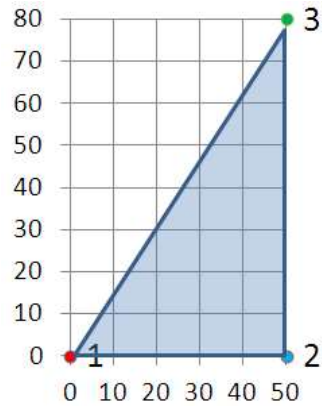
		u2	u3	v3	v4
	u2	142406.3	-16406.3	26250	0
$K =$	u3	-16406.3	142406.3	0	26250
$N \times N$	v3	26250	0	91218.75	-42000
	v4	0	26250	-42000	91218.75

		u2	u3	v3	v4
	u2	7.71864E-06	1.20993E-06	-3.02221E-06	-1.7397E-06
$K^{-1} =$	u3	1.20993E-06	7.71864E-06	-1.7397E-06	-3.02221E-06
$N \times N$	v3	-3.02221E-06	-1.7397E-06	1.53082E-05	7.54899E-06
	v4	-1.7397E-06	-3.02221E-06	7.54899E-06	1.53082E-05

	0
$F =$	0
$N \times 1$	1000
	2000



	-0.006502	mm	u2
$q =$	-0.007784	mm	u3
$N \times 1$	0.030406	mm	v3
	0.038165	mm	v4



$$B_1 = \begin{bmatrix} -0.02 & 0 & 0.02 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.0125 & 0 & 0.0125 \\ 0 & -0.02 & -0.0125 & 0.02 & 0.0125 & 0 \end{bmatrix}$$

$$q_1 = \begin{bmatrix} 0 \text{ mm} & u1 \\ 0 \text{ mm} & v1 \\ -0.006502 \text{ mm} & u2 \\ 0 \text{ mm} & v2 \\ -0.007784 \text{ mm} & u3 \\ 0.030406 \text{ mm} & v3 \end{bmatrix}$$

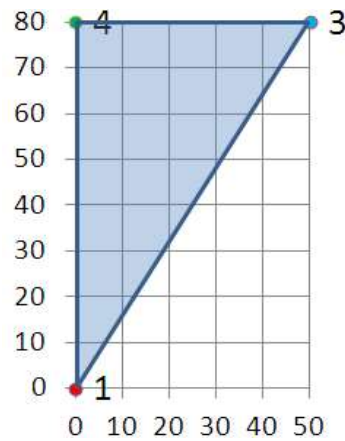
$B_1 q_1$

$$D = \begin{bmatrix} 78750 & 26250 & 0 \\ 26250 & 78750 & 0 \\ 0 & 0 & 26250 \end{bmatrix}$$

$$\epsilon_1 = \begin{bmatrix} -0.000130032 \\ 0.000380077 \\ -1.60313E-05 \end{bmatrix}$$

$D \epsilon_1$

$$\sigma_1 = \begin{bmatrix} -0.263 \text{ MPa} \\ 26.52 \text{ MPa} \\ -0.421 \text{ MPa} \end{bmatrix}$$



$$B_2 = \begin{bmatrix} 0 & 0 & 0.02 & 0 & -0.02 & 0 \\ 0 & -0.0125 & 0 & 0 & 0 & 0.0125 \\ -0.0125 & 0 & 0 & 0.02 & 0.0125 & -0.02 \end{bmatrix}$$

$$q_2 = \begin{bmatrix} 0 \text{ mm} & u_1 \\ 0 \text{ mm} & v_1 \\ -0.007784 \text{ mm} & u_3 \\ 0.030406 \text{ mm} & v_3 \\ 0 \text{ mm} & u_4 \\ 0.038165 \text{ mm} & v_4 \end{bmatrix}$$

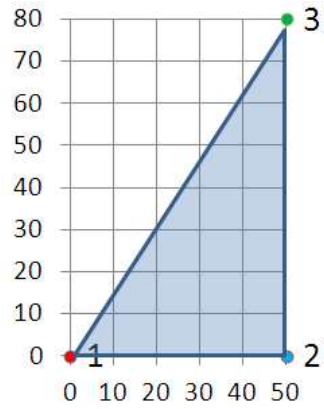
$B_2 q_2$

$$D = \begin{bmatrix} 78750 & 26250 & 0 \\ 26250 & 78750 & 0 \\ 0 & 0 & 26250 \end{bmatrix}$$

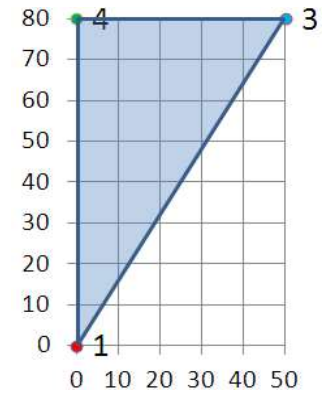
$$\epsilon_2 = \begin{bmatrix} -0.000155682 \\ 0.000477066 \\ -0.000155183 \end{bmatrix}$$

$D \epsilon_2$

$$\sigma_2 = \begin{bmatrix} 0.263 \text{ MPa} \\ 33.48 \text{ MPa} \\ -4.074 \text{ MPa} \end{bmatrix}$$



$$U_1 = 20.23940803 \text{ Nmm}$$



$$U_2 = 33.12895375 \text{ Nmm}$$

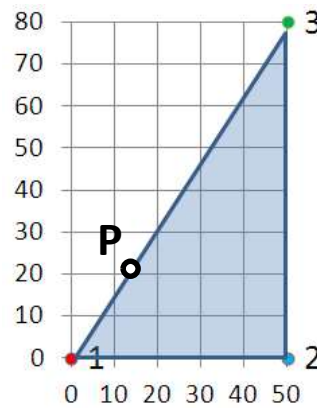
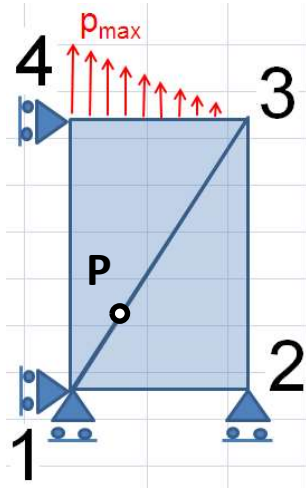


$$U = 53.37 \text{ Nmm}$$

$$U_{\text{exact}} = 59.35 \text{ Nmm}$$

$$U = 89.93\% U_{\text{exact}}$$





**Element 1**

node	$x_i$	$y_i$	$x_j$	$y_j$	$x_k$	$y_k$	$a_i$	$b_i$	$c_i$
1	0	0	50	0	50	80	4000	-80	0
2	50	0	50	80	0	0	0	80	-50
3	50	80	0	0	50	0	0	0	50

$$N_1(x_P, y_P) = N_1(12.5, 20) = \frac{a_1 + b_1 x_P + c_1 y_P}{2 \cdot A_e} = \frac{4000 \text{ mm}^2 + (-80 \text{ mm}) \cdot 12.5 \text{ mm} + 0 \text{ mm} \cdot 20 \text{ mm}}{2 \cdot 2000 \text{ mm}^2} = \frac{3}{4}$$

$$N_2(x_P, y_P) = N_2(12.5, 20) = \frac{a_2 + b_2 x_P + c_2 y_P}{2 \cdot A_e} = \frac{0 + 80 \cdot 12.5 + (-50) \cdot 20}{2 \cdot 2000} = 0$$

$$N_3(x_P, y_P) = N_3(12.5, 20) = \frac{a_3 + b_3 x_P + c_3 y_P}{2 \cdot A_e} = \frac{0 + 0 \cdot 12.5 + 50 \cdot 20}{2 \cdot 2000} = \frac{1}{4}$$

$$N_1(12.5,20) + N_2(12.5,20) + N_3(12.5,20) = \frac{3}{4} + 0 + \frac{1}{4} = 1$$

$$x = \sum_{i=1}^3 N_i(x, y) \cdot x_i \Rightarrow x_p = \sum_{i=1}^3 N_i(x_P, y_P) \cdot x_i = N_1 \cdot x_1 + N_2 \cdot x_2 + N_3 \cdot x_3 =$$

$$= \frac{3}{4} \cdot 0 + 0 \cdot 50 + \frac{1}{4} \cdot 50 = 12.5 \text{ mm}$$

$$y = \sum_{i=1}^3 N_i(x, y) \cdot y_i \Rightarrow y_p = \sum_{i=1}^3 N_i(x_P, y_P) \cdot y_i = N_1 \cdot y_1 + N_2 \cdot y_2 + N_3 \cdot y_3 =$$

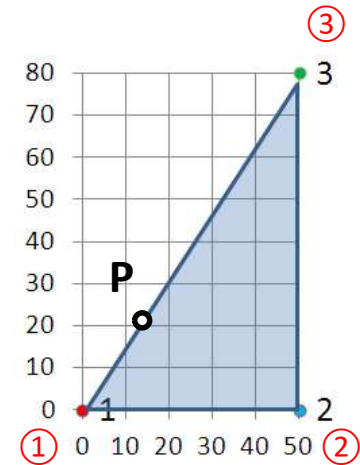
$$= \frac{3}{4} \cdot 0 + 0 \cdot 0 + \frac{1}{4} \cdot 80 = 20 \text{ mm}$$

$$u = \sum_{i=1}^3 N_i(x, y) \cdot u_i \Rightarrow u_p = \sum_{i=1}^3 N_i(x_P, y_P) \cdot u_i = N_1 \cdot u_1 + N_2 \cdot u_2 + N_3 \cdot u_3 =$$

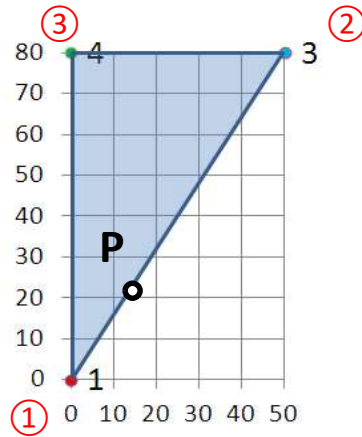
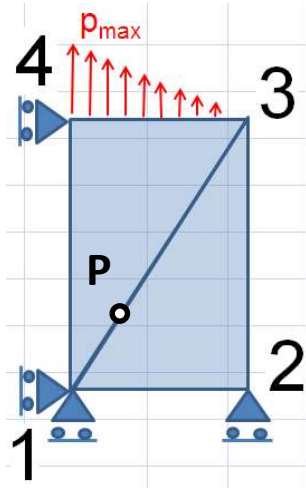
$$= \frac{3}{4} \cdot 0 + 0 \cdot (-0.006502) + \frac{1}{4} \cdot (-0.007784) = -0.00195 \text{ mm}$$

$$v = \sum_{i=1}^3 N_i(x, y) \cdot v_i \Rightarrow v_p = \sum_{i=1}^3 N_i(x_P, y_P) \cdot v_i = N_1 \cdot v_1 + N_2 \cdot v_2 + N_3 \cdot v_3 =$$

$$= \frac{3}{4} \cdot 0 + 0 \cdot 0 + \frac{1}{4} \cdot 0.030406 = 0.0076 \text{ mm}$$



	0 mm	u1
	0 mm	v1
<b>q<sub>1</sub></b>	-0.006502 mm	u2
ne x 1	0 mm	v2
	-0.007784 mm	u3
	0.030406 mm	v3



**Element 2**

node	x <sub>i</sub>	y <sub>i</sub>	x <sub>j</sub>	y <sub>j</sub>	x <sub>k</sub>	y <sub>k</sub>	a <sub>i</sub>	b <sub>i</sub>	c <sub>i</sub>
1	0	0	50	80	0	80	4000	0	-50
3	50	80	0	80	0	0	0	80	0
4	0	80	0	0	50	80	0	-80	50

$$N_1(x_P, y_P) = N_1(12.5, 20) = \frac{a_1 + b_1 x_P + c_1 y_P}{2 \cdot A_e} = \frac{4000 \text{mm}^2 + 0 \text{mm} \cdot 12.5 \text{mm} + (-50 \text{mm}) \cdot 20 \text{mm}}{2 \cdot 2000 \text{mm}^2} = \frac{3}{4}$$

$$N_2(x_P, y_P) = N_2(12.5, 20) = \frac{a_2 + b_2 x_P + c_2 y_P}{2 \cdot A_e} = \frac{0 + 80 \cdot 12.5 + 0 \cdot 20}{2 \cdot 2000} = \frac{1}{4}$$

$$N_3(x_P, y_P) = N_3(12.5, 20) = \frac{a_3 + b_3 x_P + c_3 y_P}{2 \cdot A_e} = \frac{0 + (-80) \cdot 12.5 + 50 \cdot 20}{2 \cdot 2000} = 0$$

$$N_1(12.5,20) + N_2(12.5,20) + N_3(12.5,20) = \frac{3}{4} + \frac{1}{4} + 0 = 1$$

$$x = \sum_{i=1}^3 N_i(x, y) \cdot x_i \Rightarrow x_p = \sum_{i=1}^3 N_i(x_P, y_P) \cdot x_i = N_1 \cdot x_1 + N_2 \cdot x_2 + N_3 \cdot x_3 =$$

$$= \frac{3}{4} \cdot 0 + \frac{1}{4} \cdot 50 + 0 \cdot 0 = 12.5 \text{ mm}$$

$$y = \sum_{i=1}^3 N_i(x, y) \cdot y_i \Rightarrow y_p = \sum_{i=1}^3 N_i(x_P, y_P) \cdot y_i = N_1 \cdot y_1 + N_2 \cdot y_2 + N_3 \cdot y_3 =$$

$$= \frac{3}{4} \cdot 0 + \frac{1}{4} \cdot 80 + 0 \cdot 80 = 20 \text{ mm}$$

$$u = \sum_{i=1}^3 N_i(x, y) \cdot u_i \Rightarrow u_p = \sum_{i=1}^3 N_i(x_P, y_P) \cdot u_i = N_1 \cdot u_1 + N_2 \cdot u_2 + N_3 \cdot u_3 =$$

$$= \frac{3}{4} \cdot 0 + \frac{1}{4} \cdot (-0.007784) + 0 \cdot 0 = -0.00195 \text{ mm}$$

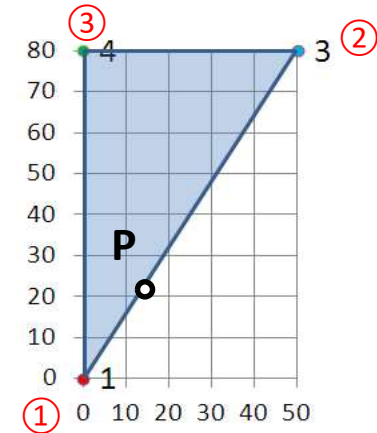
local notation

global notation

$$v = \sum_{i=1}^3 N_i(x, y) \cdot v_i \Rightarrow v_p = \sum_{i=1}^3 N_i(x_P, y_P) \cdot v_i = N_1 \cdot v_1 + N_2 \cdot v_2 + N_3 \cdot v_3 =$$

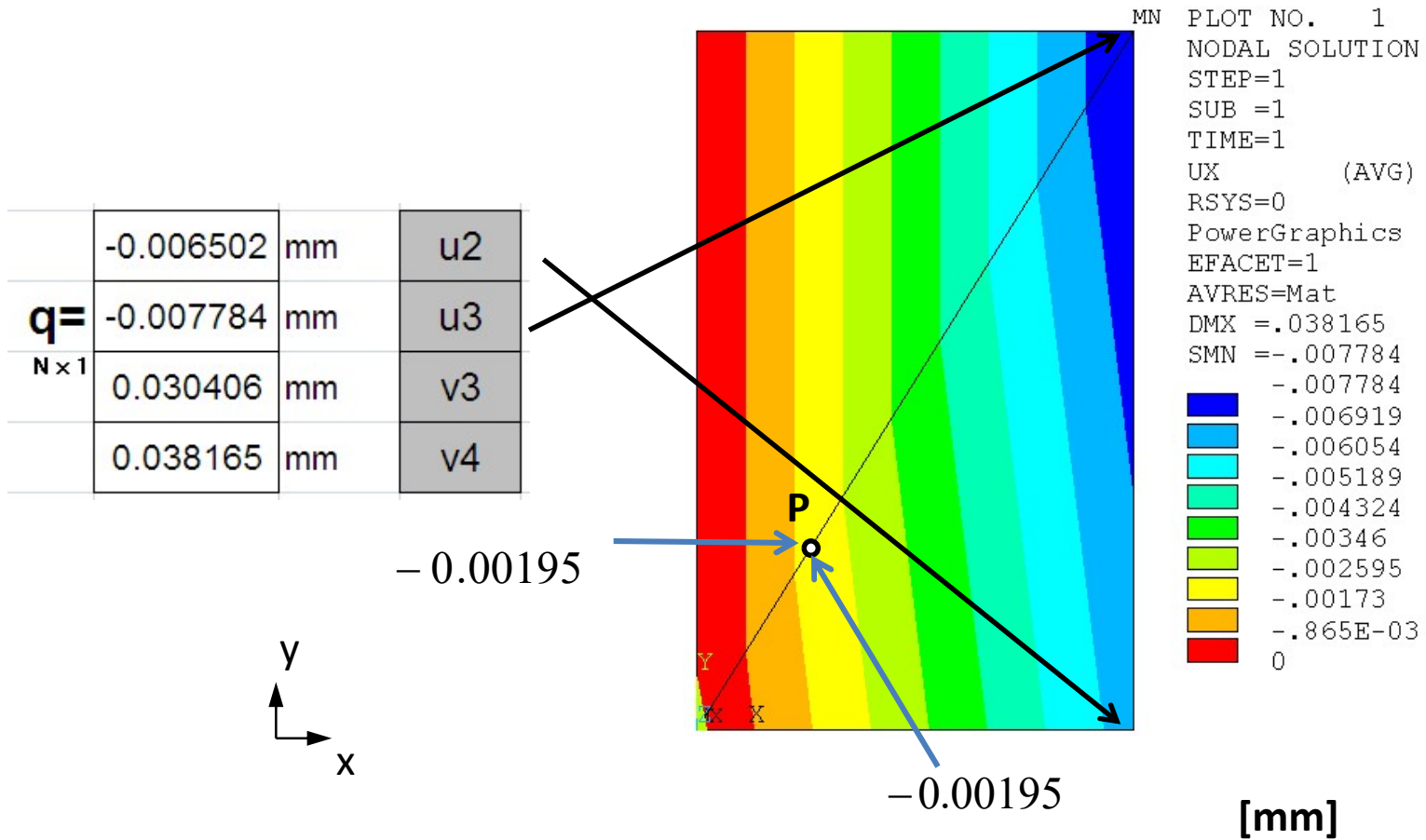
$$= \frac{3}{4} \cdot 0 + \frac{1}{4} \cdot 0.030406 + 0 \cdot 0.038165 = 0.0076 \text{ mm}$$

global notation



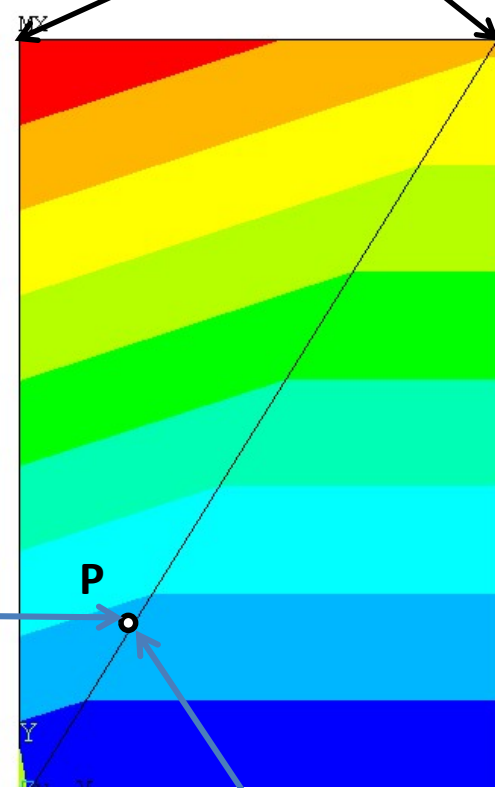
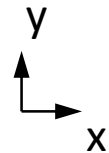
	0 mm	u1
	0 mm	v1
$q_2 =$	-0.007784 mm	u3
$ne \times 1$	0.030406 mm	v3
	0 mm	u4
	0.038165 mm	v4

# UX displacement



# UY displacement

	-0.006502	mm	u2
<b>q=</b>	-0.007784	mm	u3
<b>N x 1</b>	0.030406	mm	v3
	0.038165	mm	v4



PLOT NO. 2  
 NODAL SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 UY (AVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 AVRES=Mat  
 DMX =.038165  
 SMX =.038165  
 0

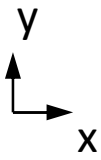
- .004241
- .008481
- .012722
- .016962
- .021203
- .025444
- .029684
- .033925
- .038165

[mm]

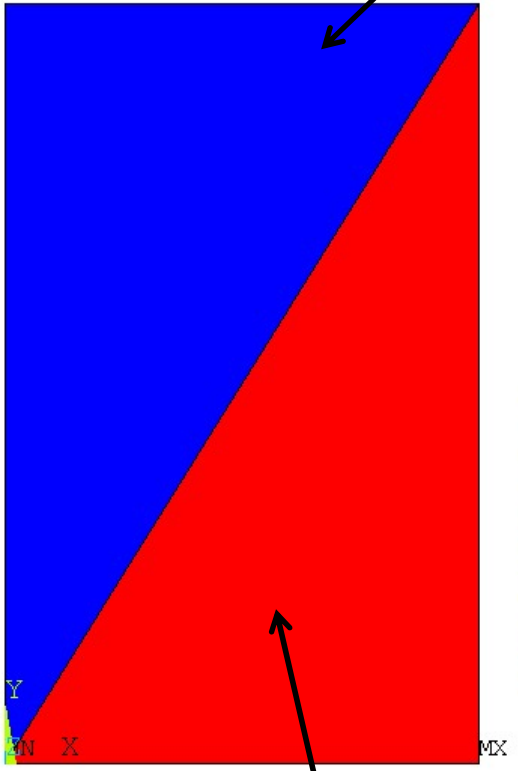
0.0076

0.0076

$\epsilon_x$  strain

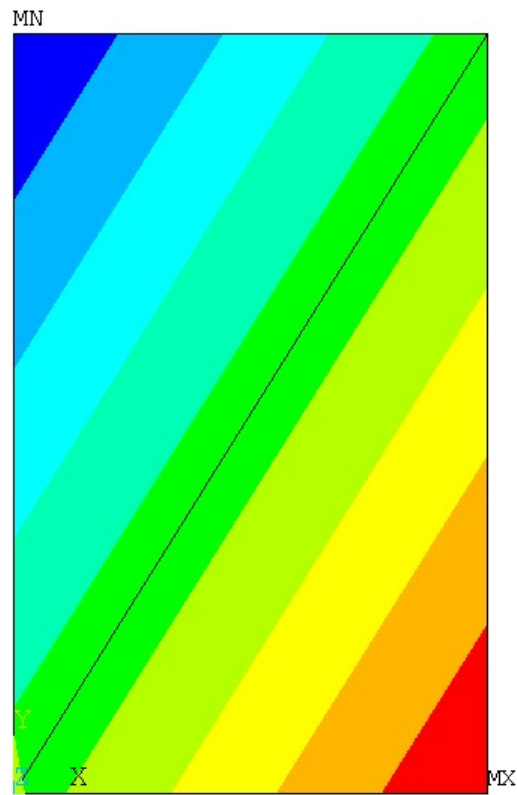


$\epsilon_2 =$	-0.000155682
	0.000477066
	-0.000155183



PLOT NO. 7  
 ELEMENT SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 EPELX (NOAVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 DMX =.038165  
 SMN =-.156E-03  
 SMX =-.130E-03

Blue	-.156E-03
Light Blue	-.153E-03
Cyan	-.150E-03
Light Cyan	-.147E-03
Green	-.144E-03
Light Green	-.141E-03
Yellow-Green	-.139E-03
Yellow	-.136E-03
Orange	-.133E-03
Red	-.130E-03

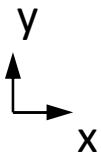


PLOT NO. 12  
 NODAL SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 EPELX (AVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 AVRES=Mat  
 DMX =.038165  
 SMN =-.156E-03  
 SMX =-.130E-03

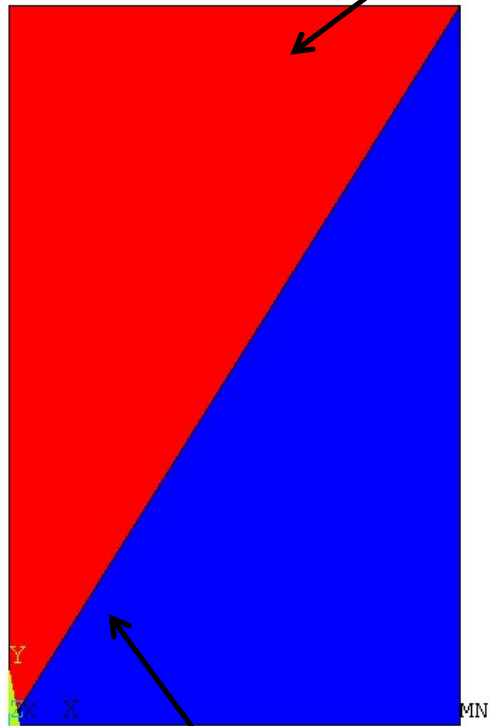
Blue	-.156E-03
Light Blue	-.153E-03
Cyan	-.150E-03
Light Cyan	-.147E-03
Green	-.144E-03
Light Green	-.141E-03
Yellow-Green	-.139E-03
Yellow	-.136E-03
Orange	-.133E-03
Red	-.130E-03

$\epsilon_1 =$	-0.000130032
	0.000380077
	-1.60313E-05

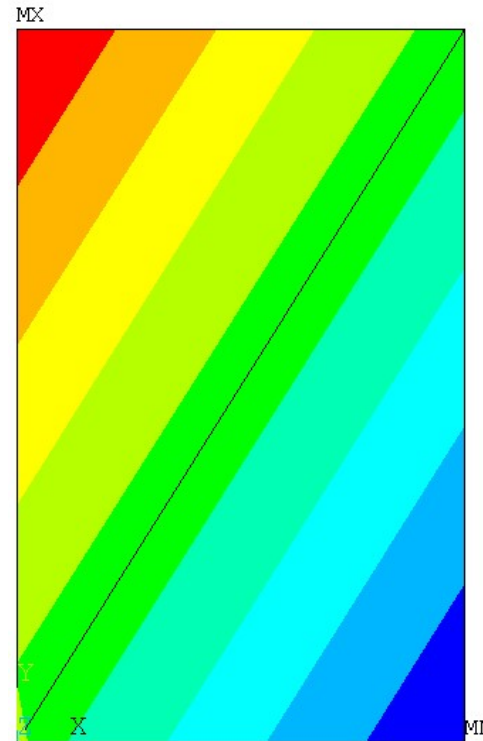
$\epsilon_y$  strain



$\epsilon_2 =$	-0.000155682
	0.000477066
	-0.000155183



PLOT NO. 8  
 ELEMENT SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 EPELY (NOAVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 DMX =.038165  
 SMN =.380E-03  
 SMX =.477E-03  
 .380E-03  
 .391E-03  
 .402E-03  
 .412E-03  
 .423E-03  
 .434E-03  
 .445E-03  
 .456E-03  
 .466E-03  
 .477E-03



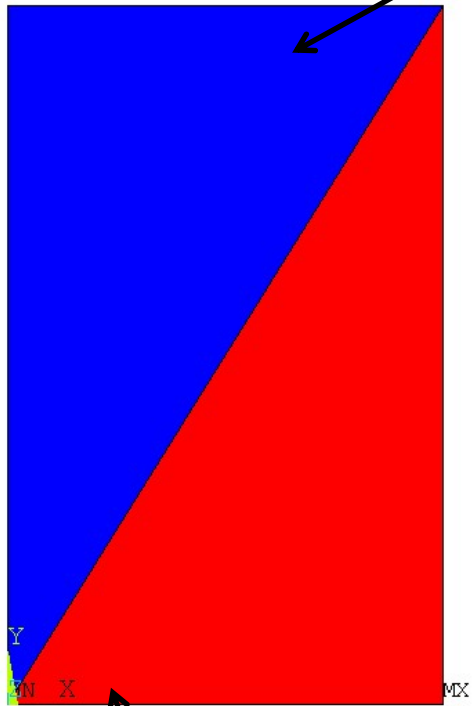
PLOT NO. 13  
 NODAL SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 EPELY (AVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 AVRES=Mat  
 DMX =.038165  
 SMN =.380E-03  
 SMX =.477E-03  
 .380E-03  
 .391E-03  
 .402E-03  
 .412E-03  
 .423E-03  
 .434E-03  
 .445E-03  
 .456E-03  
 .466E-03  
 .477E-03

$\epsilon_1 =$	-0.000130032
	0.000380077
	-1.60313E-05



$\gamma_{xy}$  strain

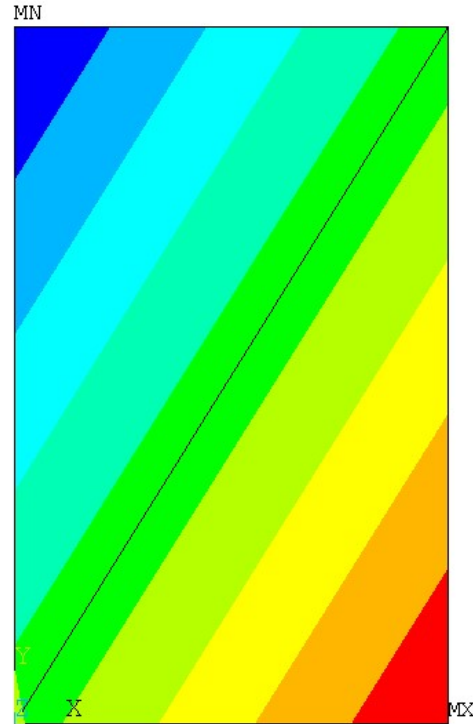
$\epsilon_2 =$	-0.000155682
	0.000477066
	-0.000155183



PLOT NO. 1  
 ELEMENT SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 EPELXY (NOAVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 DMX =.038165  
 SMN =-.155E-03  
 SMX =-.160E-04

■	-.155E-03
■	-.140E-03
■	-.124E-03
■	-.109E-03
■	-.933E-04
■	-.779E-04
■	-.624E-04
■	-.470E-04
■	-.315E-04
■	-.160E-04

$\epsilon_1 =$	-0.000130032
	0.000380077
	-1.60313E-05



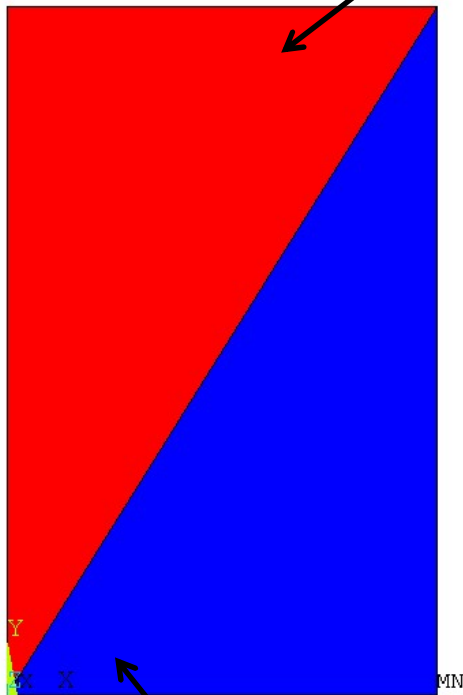
PLOT NO. 14  
 NODAL SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 EPELXY (AVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 AVRES=Mat  
 DMX =.038165  
 SMN =-.155E-03  
 SMX =-.160E-04

■	-.155E-03
■	-.140E-03
■	-.124E-03
■	-.109E-03
■	-.933E-04
■	-.779E-04
■	-.624E-04
■	-.470E-04
■	-.315E-04
■	-.160E-04

$\sigma_x$  stress

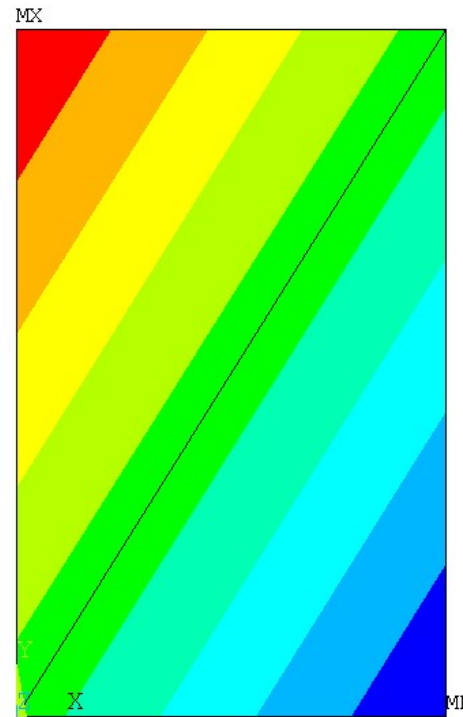


	0.263	MPa
$\sigma_2 =$	33.48	MPa
	-4.074	MPa



PLOT NO. 3  
 ELEMENT SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 SX (NOAVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 DMX =.038165  
 SMN =-.263011  
 SMX =.263011

Blue	-.263011
Light Blue	-.204564
Cyan	-.146117
Light Green	-.08767
Green	-.029223
Yellow-Green	.029223
Yellow	.08767
Orange	.146117
Light Orange	.204564
Red	.263011



PLOT NO. 9  
 NODAL SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 SX (AVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 AVRES=Mat  
 DMX =.038165  
 SMN =-.263011  
 SMX =.263011

Blue	-.263011
Light Blue	-.204564
Cyan	-.146117
Light Green	-.08767
Green	-.029223
Yellow-Green	.029223
Yellow	.08767
Orange	.146117
Light Orange	.204564
Red	.263011

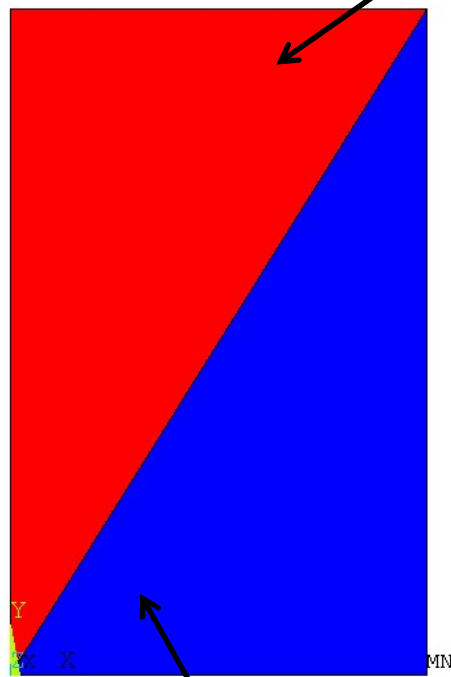
	-0.263	MPa
$\sigma_1 =$	26.52	MPa
	-0.421	MPa

[MPa]

$\sigma_y$  stress

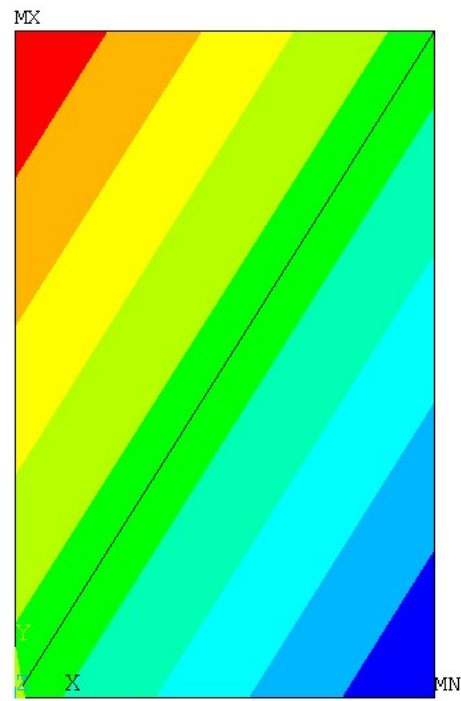


	0.263	MPa
$\sigma_2 =$	33.48	MPa
	-4.074	MPa



PLOT NO. 4  
 ELEMENT SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 SY (NOAVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 DMX =.038165  
 SMN =26.518  
 SMX =33.482

26.518
27.292
28.065
28.839
29.613
30.387
31.161
31.935
32.708
33.482



PLOT NO. 10  
 NODAL SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 SY (AVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 AVRES=Mat  
 DMX =.038165  
 SMN =26.518  
 SMX =33.482

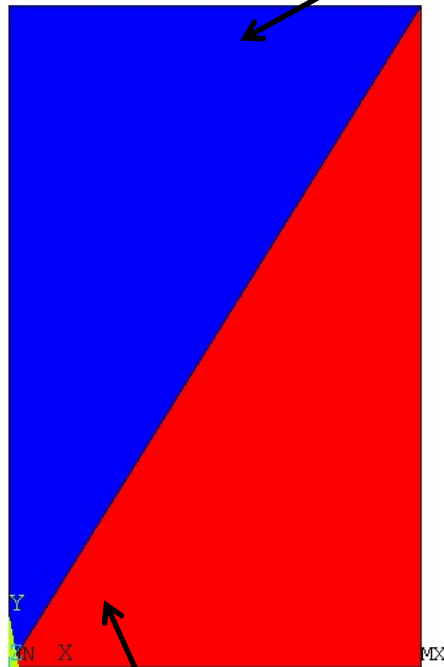
26.518
27.292
28.065
28.839
29.613
30.387
31.161
31.935
32.708
33.482

	-0.263	MPa
$\sigma_1 =$	26.52	MPa
	-0.421	MPa

[MPa]

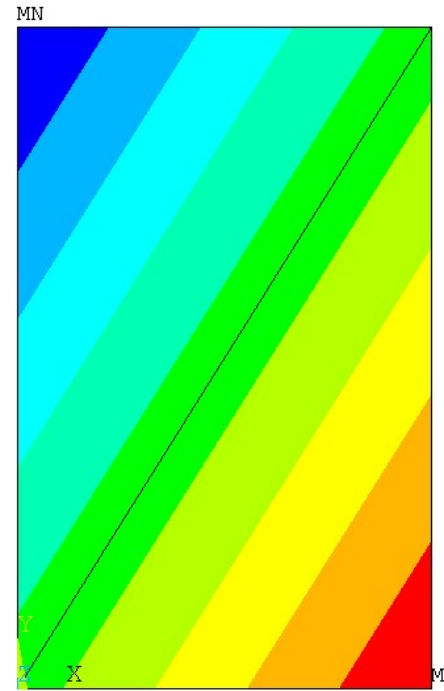
$\tau_{xy}$  stress

	0.263	MPa
$\sigma_2 =$	33.48	MPa
	-4.074	MPa



PLOT NO. 5  
 ELEMENT SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 SXY (NOAVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 DMX =.038165  
 SMN =-4.074  
 SMX =-.420818

Blue	-4.074
Light Blue	-3.668
Cyan	-3.262
Light Green	-2.856
Green	-2.45
Light Yellow	-2.044
Yellow	-1.638
Orange	-1.233
Red	-.826679
Dark Red	-.420818



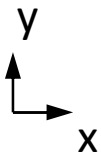
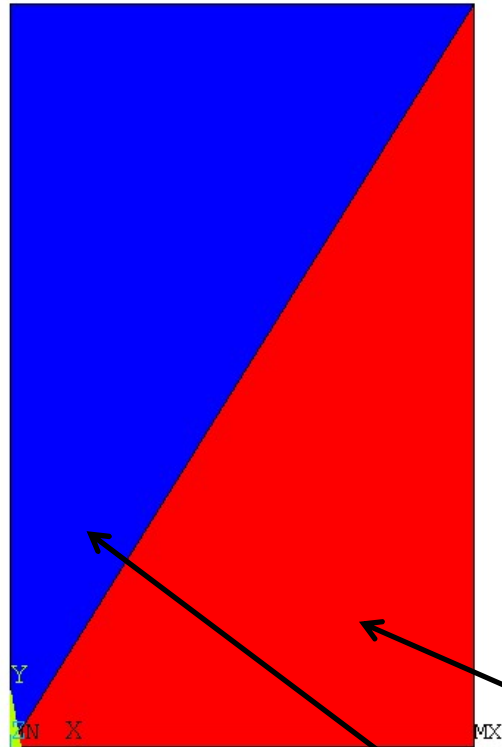
PLOT NO. 11  
 NODAL SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 SXY (AVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 AVRES=Mat  
 DMX =.038165  
 SMN =-4.074  
 SMX =-.420818

Blue	-4.074
Light Blue	-3.668
Cyan	-3.262
Light Green	-2.856
Green	-2.45
Light Yellow	-2.044
Yellow	-1.638
Orange	-1.233
Red	-.826679
Dark Red	-.420818

	-0.263	MPa
$\sigma_1 =$	26.52	MPa
	-0.421	MPa

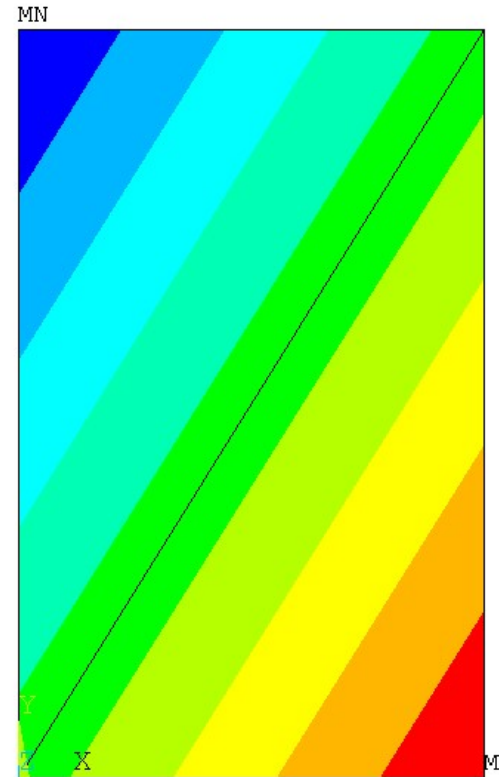
[MPa]

$\epsilon_z$  strain

PLOT NO. 6  
 ELEMENT SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 EPELZ (NOAVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 DMX =.038165  
 SMN =-.161E-03  
 SMX =-.125E-03

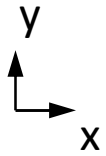
Blue	-.161E-03
Light Blue	-.157E-03
Cyan	-.153E-03
Light Green	-.149E-03
Green	-.145E-03
Light Green	-.141E-03
Yellow	-.137E-03
Orange	-.133E-03
Red	-.129E-03
Dark Red	-.125E-03



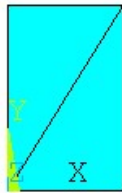
PLOT NO. 15  
 NODAL SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 EPELZ (AVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 AVRES=Mat  
 DMX =.038165  
 SMN =-.161E-03  
 SMX =-.125E-03

Blue	-.161E-03
Light Blue	-.157E-03
Cyan	-.153E-03
Light Green	-.149E-03
Green	-.145E-03
Light Green	-.141E-03
Yellow	-.137E-03
Orange	-.133E-03
Red	-.129E-03
Dark Red	-.125E-03

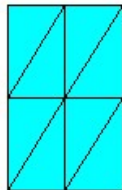
$$\epsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = \begin{cases} -\frac{1}{3.7 \cdot 10^4}(-0.263 + 26.52) = -0.125 \cdot 10^{-3} \\ -\frac{1}{3.7 \cdot 10^4}(0.263 + 33.48) = -0.161 \cdot 10^{-3} \end{cases}$$



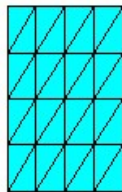
### DOF Solution



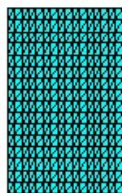
2



8



32



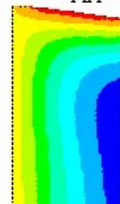
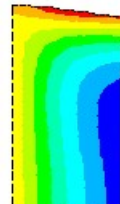
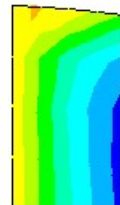
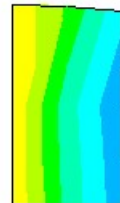
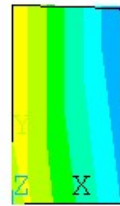
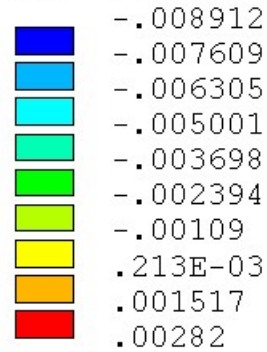
512



8000

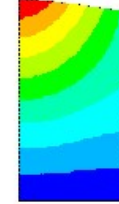
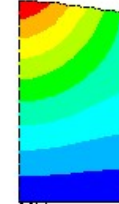
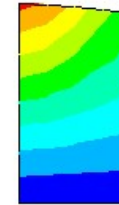
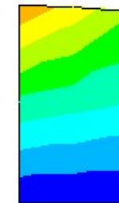
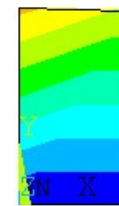
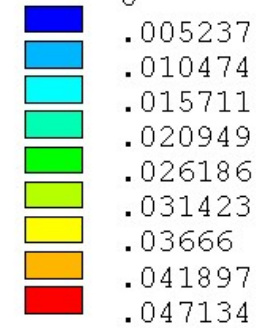
### UX

PLOT NO. 1  
 NODAL SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 UX (AVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 AVRES=Mat  
 DMX =.047134  
 SMN =-.008912  
 SMX =.00282

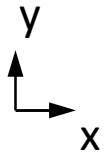


### UY

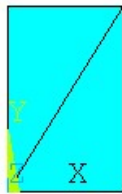
PLOT NO. 2  
 NODAL SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 UY (AVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 AVRES=Mat  
 DMX =.047134  
 SMX =.047134  
 0



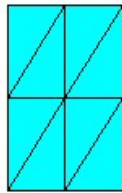
[mm]



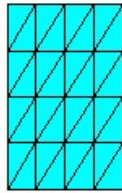
### Horizontal stress $\sigma_x$



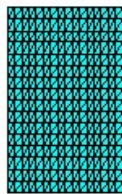
2



8



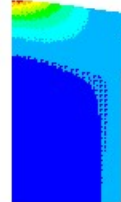
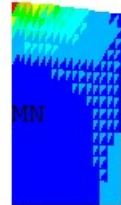
32



512

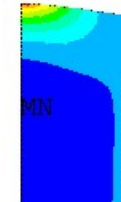


8000



PLOT NO. 3  
 ELEMENT SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 SX (NOAVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 DMX =.047134  
 SMN =-4.88  
 SMX =35.587

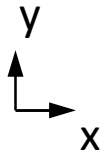
Dark Blue	-4.88
Light Blue	-.383303
Cyan	4.113
Light Green	8.609
Green	13.106
Yellow-Green	17.602
Yellow	22.098
Orange	26.595
Red-Orange	31.091
Red	35.587



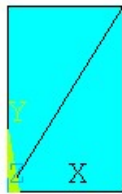
PLOT NO. 7  
 NODAL SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 SX (AVG)  
 RSYS=0  
 PowerGraphics  
 EFACET=1  
 AVRES=Mat  
 DMX =.047134  
 SMN =-4.784  
 SMX =35.587

Dark Blue	-4.784
Light Blue	-.298737
Cyan	4.187
Light Green	8.673
Green	13.159
Yellow-Green	17.644
Yellow	22.13
Orange	26.616
Red-Orange	31.102
Red	35.587

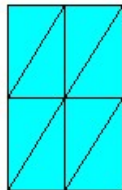
[MPa]



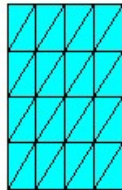
### Vertical stress $\sigma_y$



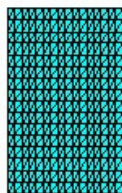
2



8



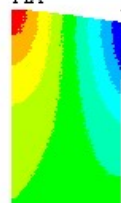
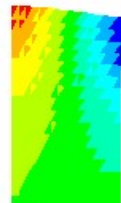
32



512

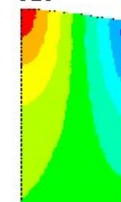
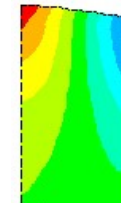
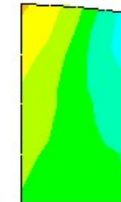
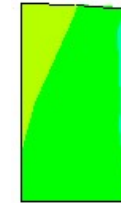
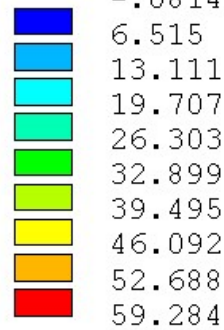


8000



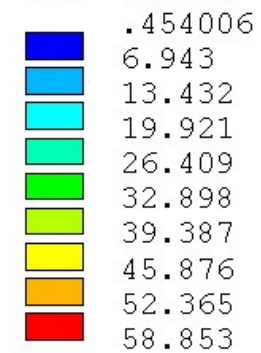
PLOT NO. 4  
 ELEMENT SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 SY (NOAVG)  
 RSYS=0

PowerGraphics  
 EFACET=1  
 DMX =.047134  
 SMN =-.081405  
 SMX =59.284

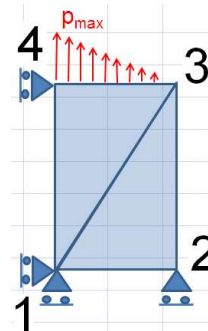


PLOT NO. 8  
 NODAL SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 SY (AVG)  
 RSYS=0

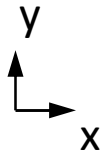
PowerGraphics  
 EFACET=1  
 AVRES=Mat  
 DMX =.047134  
 SMN =.454006  
 SMX =58.853



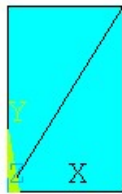
[MPa]



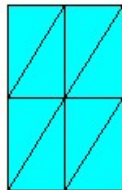




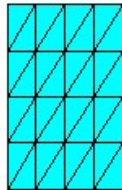
### Shear stress $\tau_{xy}$



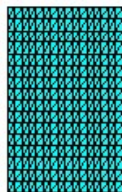
2



8



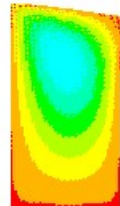
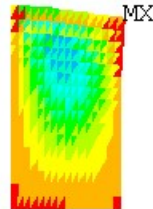
32



512

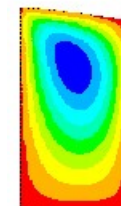
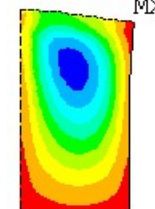
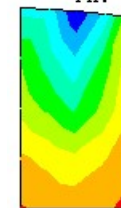
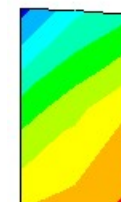
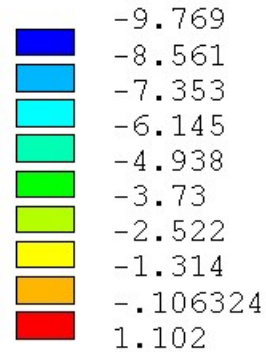


8000



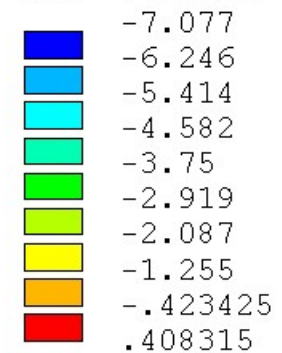
PLOT NO. 5  
 ELEMENT SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 SXY (NOAVG)  
 RSYS=0

PowerGraphics  
 EFACET=1  
 DMX =.047134  
 SMN =-9.769  
 SMX =1.102



PLOT NO. 9  
 NODAL SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 SXY (AVG)  
 RSYS=0

PowerGraphics  
 EFACET=1  
 AVRES=Mat  
 DMX =.047134  
 SMN =-7.077  
 SMX =.408315



[MPa]

